

## CHAPTER 1: Why Study Mathematics?

Those who are accustomed to judge by feeling do not understand the process of reasoning, for they would understand at first sight, and are not used to seek for principles. And others, on the contrary, who are accustomed to reason from principles, do not all understand matters of feeling, seeking principles, and being unable to see at a glance.<sup>1</sup>

For many centuries, mathematics has been regarded as an essential component of a quality education. Most people in the United States study some mathematics during the first eight years of their formal education, and typically take at least several more math courses of one sort or another during the next few years.

Everyday life seems to involve mathematics at every turn. Balance your checkbook, compute your GPA, make a double batch of cookies, figure your miles per gallon, decide how many gallons of paint you need -- all are simple matters of mathematics. One reason, then, to study mathematics is because it is useful in ordinary, every day life.

Mathematics is also absolutely essential for many careers. Engineers and scientists use mathematics constantly. Increasingly, business depends on mathematical analysis for decision-making. Many social scientists increasingly use mathematics in their work.

On the other hand, many other people seem to make little use of mathematics, and many are quite frankly happy about that fact. You can turn on your stereo and listen to music without personally using any mathematics. But someone, or more correctly, many people in our society or civilization needed to know quite a bit of rather complicated mathematics to provide that pleasure for you. On the other hand, maybe that doesn't matter to you. So if you aren't planning on a career involving the use of mathematics, why study it any more? A fair question, but one to which I believe I have a good answer. But first, consider the following.

The quote at the beginning of the chapter is from Blaise Pascal, a mathematician, scientist, and author, who lived in the 1600's. Pascal suggests that there are two ways of knowing, that one way may be more typical of a person than the other way, and that neither is necessarily better than the other. Mathematicians seem to use a methodical, step-by-step, logical process based on fundamental rules or principles to convince themselves that something is true or that they have solved a problem. That may not sound like the way you operate. If you are the kind of person whom Pascal suggests "judges by feeling", you might have reacted to some previous math class with: "I just don't think that way!" You may have been bored with the math class, or even confused by it. And you may not like the idea of taking another one.

One thing that I hope you will learn in this class is that while mathematics is often presented in a logical fashion based on fundamental principles, that's only part of the story. Mathematics is frequently developed by much more intuitive, insightful processes. In fact, mathematicians are at their best when they can combine both ways of thinking. Pascal is a good example of this. While he was able to write some first-rate, methodical mathematics, he also was very creative in the discovery phase which involved intuition. As a Christian apologist, his writings were filled with carefully reasoned arguments, but his experience with God was anything but coldly rational.

So I hope you discover that we mathematicians are not as coldly logical as you may have thought. I must confess that a while back my hero was Mr. Spock of the original "Star Trek", and so I may be guilty of perpetuating the misunderstanding. As a part of my own journey through life, I am working on the "feeling" or intuitive side of my personality so as to be more balanced. I hope that you discover that, even if your primary mode of operating is feeling, careful reasoning is important to you as well, and that this course contributes to the development of your critical thinking skills.

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<sup>1</sup>Blaise Pascal, The Thoughts of Blaise Pascal [cited hereafter as Pensees ], Doubleday, Garden City, NY, n.d., p. 9. This book is a collection of "notes" discovered in manuscript form after Pascal's death. A number of different arrangements of these "notes" exist, so finding the quotes from Pascal in a different edition of the Pensees can be difficult.

Some people might have an even more fundamental objection to this course. A person could object, on Christian grounds, to any time or energy spent on the study of mathematics. That's exactly what the following quote from Augustine, an early Church Father, a bishop of Hippo in Africa, suggests. Augustine was a very influential Christian theologian who lived around 400 A.D. He wrote this:

The good Christian should beware of mathematicians and all those who make empty prophecies. The danger already exists that the mathematicians have made a covenant with the devil to darken the spirit and to confine man in the bonds of Hell.<sup>2</sup>

Augustine certainly seems to be very clear on the matter -- drop this course before something evil happens to you! But in reality the issue is not quite so simple. Today's university catalogs tend to view mathematics as an isolated discipline, distinct from physical science, history, English literature, art, philosophy, and theology (and each of these disciplines is typically isolated from each of the others). I would suggest that Augustine held a different view, and for good reason. The people who studied mathematics in Augustine's time and place typically applied math to astrology, a practice which was condemned by Christians of that society.<sup>3</sup> In fact, the word, which in Greek or Latin looks like "mathematician", and was so translated in the above quote, is better translated "astrologer".<sup>4</sup> Augustine was not condemning mathematics as such, but rather one of its applications. As much as you might wish otherwise, I don't believe you can use this quote from Augustine to justify dropping this class.

Some twelve centuries later, Pascal wrote these words:

To speak freely of mathematics, I find it the highest exercise of the spirit; but at the same time I know that it is so useless that I make little distinction between a man who is only a mathematician and a common artisan. Also, I call it the most beautiful profession in the world; but it is only a profession; and I have often said that it is good to make the attempt [to study mathematics], but not to use our forces [i.e., try too hard]: so that I would not take two steps for mathematics, and I am confident that you are strongly of my opinion.<sup>5</sup>

It seems like Pascal vacillated between two opposite poles in his view of mathematics. On the one hand, he called mathematics "the highest exercise of the spirit" and "the most beautiful profession in the world". On the other hand, he calls it "useless" and not worth the effort of "two steps". Again, to say much more is to get ahead of our story. Pascal struggled in his own mind with the very personal question of whether to use his God-given gift of mathematical ability or whether God was calling him to do more important things with his time and energy. At the moment, it will be sufficient to recognize that you and I may have some of the same struggles. (Even if you don't see yourself as having any gift in the area of mathematics, the issue of priorities will be a real one.)

Pascal was a Christian who helped to found the mathematical theory of probability, one of the subjects we will consider later. He also used probability ideas in his apologetics. For the purpose of illustrating integration of faith and learning, it will be interesting and useful to understand him and his perspective better. Consequently, chapters will open with a quote selected from Pascal's Pensees, a collection of fragments which he intended to develop into an apologetic for Christianity. The work was incomplete when he died at age 39. In what he had written, Pascal used some of the same reasoning ability

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<sup>2</sup>quoted in Morris Kline, Mathematics for the Nonmathematician, Dover, New York, p. 1. Augustine also wrote, "For the imposters who are called mathematicians [astrologers], I did not scruple to consult (in his earlier days -ed.)... Yet true Christian piety necessarily rejects and condemns their art." (Confessions, p. 63)

<sup>3</sup>see, for instance, the reference in the Didache, 4. The Oldest Church Manual... The Didache, Philip Schaff, T & T Clark, Edinburgh, 1885.

<sup>4</sup>Bauer, Arndt, and Gingrich, Lexicon, p. 486.

<sup>5</sup>quoted in Morris Kline, Mathematics for the Nonmathematician, p. 1.

he used in mathematics to argue that believing in the God of the Bible made good sense. However, he also claimed that faith in God was a gift from God. Pascal's thinking often seems to include two opposites. But more of that later.

### The Nature of Mathematics

Morris Kline, in Mathematics for Non-Mathematicians, makes three assertions about the nature of mathematics. Since they tell us something very important about this particular author's point of view (which is also the point of view of a fair number of people today) and also introduce some important questions, we should examine them carefully.

#### 1. The origin of mathematics:

"Mathematics is a human creation." <sup>6</sup>

What does such an assertion imply? It would seem to mean that Kline believes that before there were humans, there was no mathematics. Mathematics is not something, which exists apart from human beings. It did not exist prior to human beings, waiting to be discovered by them. Rather, mathematics comes into being as a part of human culture.

Perhaps we need to take a step back before this discussion goes any further. What do we mean by "mathematics" anyway? That isn't really as easy a question as it might appear. Let me try to explain the difficulty by analogy with some other disciplines with which you are familiar. Is "history" simply a collection of "facts" like "George Washington was the first president of the United States" and "Saul was the first king of Israel"? Or is history something more, an attempt to organize the "facts" into a meaningful whole? Is a work of art merely a conglomeration of the physical materials used by the artist? Or is it something more, based on the particular way the artist has put the materials together? Chemistry involves the study of the elements, like oxygen, hydrogen and iron. But chemistry is really theories about how the elements work, how they combine, why they have the properties they do, etc.

In many people's minds there are simple mathematical "facts"; for example, " $2 + 2 = 4$ " and "The sum of the angles of a triangle is 180 degrees." For Kline, mathematics means more than just a collection of such "facts". He means a logical system which combines such "facts"; for instance, Euclidean geometry or the whole subject of number theory. Kline is saying something like: "chemistry is a subject which humans created" or "da Vinci created 'The Last Supper'." These statements would not necessarily contradict the fact that God created the elements or that da Vinci was using God-given talent to paint with materials created by God. But they would say that our current theories of how elements combine to make molecules, etc., are only human understandings, and that da Vinci did something creative with what God gave him. So Kline may not be making an assertion about the source of the "facts" at all, just about the systems.

Now some people would assert that even the "facts" of mathematics have been made by humans. On the other hand, many people would see the "facts" of mathematics, like " $2 + 2 = 4$ ", to be truths which we have merely discovered. Then the question becomes, where did these truths come from? Maybe they are truths which have always existed:  $2 + 2$  simply just is 4, and could not ever have been otherwise. What do you think about that?

Kline does not claim to be a Christian, or even a theist. He does seem to write from a humanist point of view. His understanding of the origin of mathematics includes no role for God. However, that doesn't automatically mean that he has nothing valuable to say. As with any author, we need to critically examine his ideas. And since he has written a number of popular books about mathematics, we shall consider his ideas on several occasions.

Would a Christian alternative to Kline's assertion be that mathematics is a creation of God? Certainly in some sense God is the Creator of all that exists. (Well, not evil. And while God created Satan as an angel, God isn't the creator of Satan as Satan. So, maybe the issue isn't quite as easy as it might first

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<sup>6</sup>ibid., p. 3

appear.) Augustine wrote, "numbers. . . have fixed laws which were not made by man, but which the acuteness of ingenious men brought to light."<sup>7</sup> While Augustine didn't go into great detail, I would read this as an assertion that God made both the "facts" and the systems ("laws") of mathematics (or at least arithmetic). Is that the one and only truly Christian position?

Is God the creator of all that we call mathematics today? I'm a little hesitant to immediately say a simple yes. At this point I would merely suggest you think of some other questions which may shed some light on some of the issues involved. Did God create the boysenberry? If you haven't been to Knott's Berry Farm, or you went there just for the rides, you might not know that "in 1927, a neighbor showed Walter Knott how to cross three berries to get a luscious new fruit, the boysenberry," according to the jar of preserves in my kitchen. At least according to the Knott's, Walter Knott was the creator of the boysenberry back in 1927. So is the boysenberry man-made, in the sense that a man used the mind that God gave him to work with three parts of the world God made to bring into being something new? Is mathematics like the boysenberry in any way?

Or, thinking in a different direction, what about Beethoven's Ninth Symphony? Did Beethoven create it? Are human beings, made in the image of God, able to create music? Or, is all music created by God? Did Beethoven just "discover" a tune God had been humming for millennia? Is mathematics like boysenberries and symphonies, or is it different somehow? If so, how?

## 2. The methodology of mathematics:

"Mathematics is concerned primarily with what can be accomplished by reason."<sup>8</sup>

Now there are different kinds of reason, and we will discuss these in detail later. For now it will suffice to say that Kline sees the use of deductive reasoning to be central to the nature of mathematics. Deductive reasoning uses rules of logic to derive conclusions which are certain to be true if you start with true statements. No one who has taken a geometry class would disagree with the claim that mathematics uses this kind of reasoning. But so do other disciplines. How does the methodology of mathematics differ from the methodology of subjects like philosophy, psychology, theology, or politics?

Notice again Kline's focus on the role of the human mind. Perhaps this is the time to at least ask this question: did the sin of Adam and Eve in the Garden of Eden have an effect on the mind of humankind? If so, does that mean that sin affects mathematics today? How? Would a Christian view of mathematics involve some sort of redemption, some attempt to remove or overcome the effects of sin?

## 3. The purpose of mathematics:

"The primary objective of all mathematical work is to help man study nature ..."<sup>9</sup>

For Kline, the connection between mathematics and science (primarily physics and astronomy) is very strong. And for good reason, as our historical approach to the study of mathematics will show. Up until rather recent times, mathematics has seldom been viewed as an isolated discipline to be developed apart from science. So for the Christian who would adopt Kline's perspective at this point, the question would become, "What justification does a Christian have for studying nature?" Psalm 19:1 states, "The heavens are telling of the glory of God..." For many people, they would experience this glory by seeing a sunset or a rainbow. If, however, they had to look at the mathematical formulas explaining the colors, the experience might lose its glory.

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<sup>7</sup>Augustine, "On Christian Doctrine", Great Books, p. 654.

<sup>8</sup>ibid., p. 3

<sup>9</sup>ibid., p. 5

On the other hand, one could argue that mathematics exists as an independent entity, and need not serve science to justify its existence. Then what is its goal? Is doing mathematics like climbing the proverbial mountain: you do it because it is there? Is it justification enough to prove a theorem merely because someone thought about it?

We should also note that "nature" for Kline includes human nature. Those of you who are interested in psychology will find that the development of mathematics gives us some insight into the workings of the human mind. Kline is suggesting that we may understand ourselves better by reflecting on this product of our minds called mathematics.

Perhaps as you have read this, the thought crossed your mind that the nature of mathematics could have been discussed more simply by naming the objects of mathematics. Mathematics is about numbers and shapes, right? Well, yes it is, and we will spend a large portion of this class talking about numbers and shapes. The trouble is, there isn't quite as much consensus as you might think on what numbers and shapes are. In the case of numbers, the very names we have given some of them suggest the uncertainty about their nature: negative numbers, irrational numbers, imaginary numbers. Oh, you may be so used to those names that the connotations don't hit you. But think about it: what are irrational numbers doing in a subject which, to quote Kline, "is primarily concerned with what can be accomplished by reason"? And what about "imaginary numbers"? Is mathematics reasonable if it is only about things that we are pretending exist? Does -1 have a square root or doesn't it?

In the case of shapes, did you know that there are some ways of doing geometry in which rectangles do not exist? Before you stop reading, think about it this way. Maybe it's like playing basketball with a three-point shot. That wasn't the way it used to be when I was in college. So is this "new" game still basketball, or is it something else? I think both games are basketball. And some people think mathematics is just a bunch of games. Confused? Well, stick around and we'll try to sort it out.

### The Values of Mathematics

Morris Kline suggests that there are six reasons that mathematics should be valued as a part of culture. These contributions to our society can be recognized even if we cannot agree at the moment on the exact nature of mathematics. An additional question would be, are these values that a Christian can believe in?

1. Mathematics has practical value.

This is the use of mathematics as it relates to technology as well as every day life. Even the person who has no interest in the details of technology has some appreciation for the contribution it makes to our quality of life. While there is some reason for critiquing a technological view of life, there are definite benefits of technology to which mathematics has contributed significantly.

2. Mathematics helps us understand nature for its own sake.

In contrast to technology which focuses on applications, this is science. Our interest here will be on questions one might ponder on a clear night: how do the movements of the sun, the moon, the planets and the stars make sense? And how big are they, and how far away?

3. Mathematics has helped to de-mystify nature.

For early human beings, eclipses must have been mysterious events. Even before a complete explanation was available, mathematics helped to take some of the mystery away by allowing eclipses to be predicted. Now the mathematics of astrophysics describes them in complete detail.

For Kline, an important function of modern science was the complete removal of the need for "mysterious" explanations for phenomena observed in nature. There is room for quite a bit of debate on this

point. Is there no more room for miracles? Does this deny the doctrines of Creation and Providence? Suffice it to say at the moment that science has at least removed some occurrences from the realm of "mystery", although maybe not quite as many as some people would have us believe.

#### 4. Mathematics helps us in the study of human nature.

Since mathematics utilizes reasoning in such a fundamental way, studying mathematics and its practitioners gives us insight into the workings of the human mind. In particular, mathematicians see themselves as needing to be very creative. Many non-mathematicians (artists, musicians, poets, . . .) are involved in activities which are more traditionally associated with creativity. How do these two facets of creativity relate to one another?

#### 5. Mathematics raises and informs the discussion of philosophical issues.

Kline notes that "perhaps the most profound questions in the realm of philosophy are, What is truth and how does man acquire it?"<sup>10</sup> For two thousand years, philosophers grappled with how to understand "2+2=4" and its relationship to other "truth". Descartes, the father of modern philosophy, found in mathematics the method by which he thought all truth was to be established. We shall discuss this in more detail later. On the other hand, the last 150 years have seen a revolution in mathematics and its philosophical "implications".

#### 6. Mathematicians search for beauty.

Now I'm sure some of you are saying, "He's got to be kidding!?" If there is one adjective you probably would not have thought would relate to mathematics, it is "beautiful". Art is beautiful, music is beautiful, a woman is beautiful, a sunset is beautiful,.... But math is beautiful?? Well, "beauty is in the eye of the beholder." So I may not convince you that mathematics should be beautiful to you, but I would like to convince you that I and others like me find it beautiful, and therefore enjoyable, and that any thorough discussion of beauty will need to include mathematics.

Mathematicians have also noticed that beautiful music and beautiful art often have some mathematical pattern or relationship underlying them. I'm not sure how to explain all of that, but the math seems to be there.

### Central Themes

There are several themes which will appear as this course develops. Let me introduce one at the beginning.

The following quote from Pascal appears on the title page: "All things proceed from the Nothing, and are borne towards the Infinite."<sup>11</sup> Infinity is a concept which we will encounter many times in this course. Some mathematicians see infinity as the central notion of mathematics. It is also true that God is often described as infinite.<sup>12</sup> I do not believe that "infinity" in mathematics is the same as "infinity" in theology. However, I am inclined to think that there is a sense in which "infinity" in mathematics is a "reminder" for us about God. God has graciously revealed truth about Himself to us, and I think "infinity" in mathematics may be another part of that revelation. See what you think as we proceed.

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<sup>10</sup>ibid., p. 7.

<sup>11</sup>Pascal, Pensees, p. 27

<sup>12</sup> For instance, see Louis Berkhof's discussions of "The Infinity of God" in Systematic Theology, Eerdmans, Grand Rapids, MI, 1979, pp. 59-61.

### Different Views of Mathematics

What is mathematics? When asked that question, most people respond that mathematics is a **tool** used to solve problems. In other words, mathematics must be *practical*, whatever that means. But is that the only way of viewing mathematics? One of the objectives of this course is to encourage you to view mathematics in some other light besides that of a tool. Indeed, some mathematicians are even surprised that mathematics is practical at all. Tony Chan, UCLA Professor of Mathematics, when interviewed by a reporter for the L.A. Times in 2000 said,

“Mathematics has a strange gift. It is “unreasonably effective” in places where it has no right to be.”

So we challenge you. During the course of the semester, make an effort to identify those instances when mathematics has been presented as something other than a tool. Perhaps it has been used as a language, a system of logic, an art form or even a game. You might also try to identify the primary way that your instructor identifies with mathematics.

### CHAPTER 1: Why Study Mathematics

### Homework

1. Write a couple sentences about your major or discipline describing its origin, methodology, and purpose from a non-Christian perspective. Then write a couple sentences describing its origin, methodology, and purpose from a Christian perspective. Explain the similarities and/or differences.
2. List some of the reasons your major or discipline is valuable.
3. State in a sentence or two your belief about whether human beings create or discover mathematics, or provide your own alternative.
4. State in a sentence or two your belief about how sin has (or has not) affected the reasoning ability of human beings. In a sentence or two, state the implications of this for mathematics. What about the implications for your major?
5. In Pascal’s terminology, would you consider yourself more of a “feeling” person or a “reasoning person”? Explain.
6. State in a paragraph what you think is meant when God is described as “infinite”.