CHAPTER 2: A Historical Orientation

> Jesus Christ is the end of all, and the center to which all tends. Whoever knows Him knows the reason of everything.[13]

Math Influences Culture

In an article entitled, "The Centrality of Mathematics in the History of Western Thought", Judith Grabiner has written,

> ... history shows that the nature of mathematics has been–and must be–taken into account by anyone who wants to say anything important about philosophy or about the world.... ...one cannot fully understand the humanities, the sciences, the world of work, and the world of man without understanding mathematics in its central role in the history of Western thought. [14]

It is the contention of this historian of mathematics that, at least in western civilization, mathematics has played such an influential role that it has affected most every other discipline. In some cases, like physical science, that should seem obvious. But in other cases, such as sociology or literature or music or art, the connection may not be as clear. In fact, you may even doubt that a person who was interested in understanding one of these disciplines would have any concern about mathematics. I hope this course will convince you that there are many more significant connections than you might ever have guessed.

These connections are a part of what integration of faith and learning is all about. Since "all truth is God's truth", it should not be surprising that it is difficult (if not impossible) to find the truth of one discipline in isolation from the truth of other disciplines. One discipline which Grabiner does not mention is theology. Yes, mathematics has influenced even theology. I hope you will find that this course presents at least one model of how integration of faith and learning can be pursued.

Culture Influences Mathematics

Why did mathematics develop the way it did? Raymond Wilder has written,

> Mathematics is something that man himself creates, and the type of mathematics he works out is just as much a fact of the cultural demands [I would say, "values"] of the time as any other of his adaptive mechanisms.[15]

In his book, Wilder suggests that there are two types of influences, or forces, which affect the development of mathematics in a particular time and place. To what degree these various factors are important or are given emphasis influences the style of mathematics which will develop in that culture.

First, Wilder suggests there are forces which are internal to mathematics. These are the values and concerns of mathematicians themselves as a sub-culture. Consider the following examples. Generalization or abstraction may or may not be of concern to mathematicians of a particular time and place. Numbers may be thought of quite concretely, or they may be treated as abstract concepts. The distinction between an exact solution to a problem versus an approximate solution is important to some people; other people couldn't care less. To some people, a statement or a formula would only become a part of mathematics after it has been proven; more pragmatic mathematicians might use it if it seems to work adequately. To some people, making sure that mathematics is solidly grounded in fundamental principles is of vital importance; to others, such concerns would be of little consequence. I have already

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[13]Pascal, <u>Pensees</u>, p.184.

[14]Judith Grabiner, "The Centrality of Mathematics in the History of Western Thought", <u>Mathematics Magazine</u>, Vol. 61, No. 4, Oct. 1988, p. 229.

[15]Raymond Wilder, <u>Evolution of Mathematical Concepts,</u> Halsted Press, New York, 1975, p. 3.

mentioned above that mathematics has an aesthetic aspect; how much this feature of mathematics is stressed may vary. Finally, there are specific concepts or procedures which mathematicians view differently. One such concept which we will discuss at some length is the concept of infinity. At the moment it is sufficient to note that, until recently, most mathematicians thought that infinity had no place in mathematics.

Wilder also sees the development of mathematics as being affected by forces which are external to it. These are the attitudes and values and concerns of the culture as a whole. An example in this category would be the value the society places on learning. A related issue would be the nature and focus of education: is it training for a vocation or something else? Philosophical and religious values would have an influence on mathematics. Another very significant external influence would be the way mathematicians are supported financially. Some governments support extensive work by mathematicians for economic purposes or for military purposes. Who signs their checks may influence what kinds of problems mathematicians try to solve!

Early Civilizations

It is important to note from the outset that a person like Morris Kline approaches the history of mathematics from a perspective based on his views of the nature of mathematics. It is clear that he places a very high value on the use of deductive reasoning to establish the validity of mathematical results. Since this use of deductive reasoning is not evident in early civilizations, Kline does not have much regard for the mathematics of these cultures. He claims, "very few [early civilizations] possessed any mathematics worth talking about".[16] It may not meet Kline's standards, but there is certainly some mathematics in these cultures which is worthy of our consideration. We will learn a few aspects of the mathematics of the Egyptians and Babylonians which are of significance for our later considerations.

Egyptian Mathematics

The Egyptian numeral system (the method or style of writing numbers) was based on the number 10, just like our system. Unlike our system, however, different powers of 10 (hundred, thousand, ten thousand,...) had distinctly different symbols and the value of the symbol was not dependent on its place. That is, their system was not positional. (Our system is positional in that the symbol "2" has a different value in "23" than it has in "32" because it is in a different position.)

The Egyptian system for multiplication is quite different from ours. Our system is based on repeated use of the times tables up to 9 X 9. This requires extensive memorization. The Egyptian system is based on repeated doubling (adding a number to itself) and addition of some of the results. This eliminates the need to memorize any times tables!

First, let me describe the Egyptian system in words. It is clearly based on the fact that multiplication of m • n is the addition of m n's; we will always interpret the first number as how many of the second number we are adding. For example, 7 • 8 = 8 + 8 + 8 + 8 + 8 + 8 + 8 = 56. But to streamline the process, the Egyptians used the technique of repeated doubling; in this example, we would add 8 + 16 + 32 = 56. The efficiency of this approach becomes more evident when the numbers are larger.

Here are the details we will write to use the Egyptian process: to multiply 7 times 8, one 8 is 8, two 8's is 16, and two 16's (which is the same as four 8's) is 32. So seven 8's would be 8 + 16 + 32 = 56. In more typical form (where* indicates numbers added to get seven and then seven 8's which is the final answer)

[16]Kline, Mathematics for the Nonmathematician, p. 11.

7 x 8:	1*	8*	
	2*	16*	
	+4*	+32*	
	7	56	8 + 16 + 32 = 56

Example: 9 x 7 would be done by doubling 7 to get 14 (two 7's), doubling 14 to get 28 (four 7's), and doubling 28 to get 56 (eight 7's). In more typical form, the problem would appear like this:

9 x 7:	1*	7*	
	2	14	
	4	28	
	+8*	+56*	
	9	63	7 + 56 = 63

Example: 11 x 6 would be done as follows:

11 x 6:	1*	6*	
	2*	12*	
	4	24	
	+8*	+48*	
	11	66	6 + 12 + 48 = 66

Multiplication by a "two-digit" number is handled without any new procedures. Even if two "two-digit" numbers are involved, the same procedure is used.

Example: 13 x 24:	1*	24*	
	2	48	
	4*	96*	
	+8*	+192*	
	13	312	24 + 96 + 192 = 312

Example: 21 x 17:	1*	17*	
	2	34	
	4*	68*	
	8	136	
	+16*	+272*	
	21	357	17 + 68 + 272 = 357

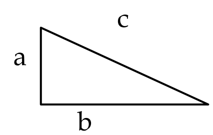
Egyptian geometry was a collection of procedures for finding quantities like areas and volumes. Sometimes the procedures they used seem to lead to correct answers, sometimes merely more-or-less adequate approximations. The distinction between exact and approximate seems to be one which the Egyptians would not have made. There were no formulas in our sense of the term. And the procedures don't seem to have been logically derived or connected to one another. In other words, Morris Kline would not call Egyptian geometry real "mathematics". On the other hand, they made significant advances, and provided some of the building-blocks from which Euclidean geometry would be built by the Greeks.

Babylonian mathematics

The Babylonian numeral system was based on 60. While this may seem very strange, there are good reasons why this led to a much better system than the Egyptian system when dealing with fractions. It was carried over by the Greeks for use in astronomy, and is still used today in angles (degrees are sometimes split into 60 minutes) and time (60 seconds in a minute, 60 minutes in an hour).

Babylonian geometry focused on numerical relationships between objects. While formulas as we know them were not expressed, tables of specific number relationships suggest that the Babylonians knew patterns. One such pattern is the (so-called) Pythagorean Theorem: in a right triangle, the sum of the squares of the legs of the triangle equals the square of the hypotenuse (the side opposite the right angle). In mathematical notation, the pattern looks like this:

$$a^2 + b^2 = c^2.$$



Example 5: The numbers 3, 4, and 5 form a Pythagorean triple because they fit this pattern:

$$3^2 + 4^2 = 5^2 \quad \text{or} \quad 9 + 16 = 25.$$

Example 6 : The numbers 5, 8, 10 do not form a Pythagorean triple because $5^2 + 8^2 = 89$, but $10^2 = 100$.

Example 7: There is no triple that has 4 and 5 as its first two numbers, because $4^2 + 5^2 = 16 + 25 = 41$, and 41 is not the square of any (whole) number. My calculator says: $\sqrt{41} = 6.403124237$.

In summary, pre-Greek mathematics was based on inductive reasoning and illustrated with concrete examples. General rules, abstract concepts, proofs and formulas as we know them did not exist. Some problems were solved exactly, while others were only solved with approximation. More significantly, the distinction between exact and approximate seem to have been unclear.

Greek Classical Period 600 - 300 BC

Since we will deal with this period in some detail later, we will make just a few general observations at this time. First, it was during this period that abstract concepts and general rules first appear in the Mediterranean area. The deductive method was developed, and utilized as a way of reasoning which leads to certainty about the results. While approximate answers are sometimes very useful, it became an important issue to distinguish between approximations and exact answers. And although answers are very important, the Greeks also sought for first causes. In mathematics, these foundational principles were the axioms. The culmination of all of these characteristics was Euclid's Elements, which organized the geometry and number theory known at the time.

Later Greeks 300 BC - 500 AD

During this period Euclidean geometry was developed further, and applied in significant ways. The greatest and most well-known mathematician of the time was Archimedes.

It is common during this period to find mathematics, philosophy, astronomy, and astrology all linked together. The quote from Augustine in Chapter 1 in which he condemned "mathematicians" was from the last century of this period. An infamous incident which occurred about the same time was the stoning of Hypatia. Hypatia is known as the first woman mathematician. She taught mathematics (and philosophy) at Plato's Academy, a school which was pagan at a time when the Roman Empire had been officially "Christian". The way the story is often recounted, she was seized by a "Christian" mob on day after a lecture, was dragged into the streets, and stoned. Opponents of Christianity sometimes cite this example today to illustrate what they see as the anti-intellectualism of Christianity. They accuse the Church of causing the decline of mathematical learning. There are several questions which are often ignored in the citing of this story. Was Hypatia opposed for her mathematics, or her anti-Christian philosophy? Was the mob really composed of Christians, or merely people who were Christians because all inhabitants had been declared Christian? On the other hand, we as Christians should ask ourselves the question: what measures are legitimate for us to use today in our opposition to the influence of non-Christians or non-Christian ideas?

Romans

While the Romans were very good engineers, they did little to develop mathematics. Apparently mathematics was not highly valued in Roman culture, at least not the way the Greeks valued it. The Roman statesman Cicero (106-43 B.C.), a contemporary of Julius Caesar, wrote: "the Greeks held the geometer in the highest honor; accordingly, nothing made more brilliant progress among them than mathematics. But we [Romans] have established as the limit of this art its usefulness in measuring and counting."[17] Do you know anything about the Romans that would help us understand this "limit" on mathematics? What aspects of culture or human activity did the Romans value? In what areas does Roman culture continue to impact our culture today?

China

The Chinese were quite proficient in performing calculations and solving practical problems. Books were written to train the users of mathematics as early as 100 BC. In some cases, mathematical results (such as the so-called "Pythagorean theorem") were discovered earlier in China than they were discovered in Europe. However, there was no development analogous to the deductive method of the Greeks. We will consider the mathematics of China in a subsequent chapter.

500 - 1300 A.D.

European Middle Ages

Relatively little mathematics was even preserved in Europe during this time. We will have nothing more to say about it.

Hindus

In what is now India, the Hindu culture developed significant number theory based on earlier results. The forerunners of the symbols 1,2,3,...9 were being introduced and refined during this period, and zero and negative numbers were being recognized as legitimate numbers. We will look at some details in a later chapter. Other more advanced problems were also being solved.

Arabs

The Arabs were the preservers of Greek mathematics while it was lost in western Europe. What they learned from the Hindu culture to the east, preserved from the Greek culture to the west, and

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[17]Cicero, Tusculan Disputations, J.E. King, trans., Harvard Press, 1951, p.7.

developed themselves, was later passed to Europe, and was foundational to the development of mathematics in the Renaissance. We will say more about their role in a subsequent chapter.

Renaissance Europe 1300 - 1550 AD

Interest in mathematics, as with other aspects of culture, was reborn in Europe during the period known as the Renaissance. There are many significant aspects of life which had important influences on the development of mathematics and vice versa. We will discuss some of these in more detail later; at this point the list serves to illustrate how integrated mathematics was with other parts of the culture:

art (perspective)
astronomy (motion in the heavens, like planets)
war (motion of projectiles, like cannonballs)
travel (navigation across oceans)
printing (dissemination of all types of knowledge)
the Protestant Reformation.

During this time, Euclidean geometry was rediscovered, and new approaches to geometry were explored. Algebra, which had been significantly improved by Hindu and Arabic contributions, began to develop as a separate branch of mathematics, independent of geometry. In general, mathematics became an important tool.

Mathematics and Modern Science 1550-1800 AD

While the basics of algebra were known earlier, this period saw the introduction of much of the notation we associate with algebra today. This increased the ability of users of algebra to apply it more effectively to a wider array of problems.

Geometry and algebra were linked in what is called analytic geometry or coordinate geometry. This made many geometry problems easier to solve.

Both algebra and analytic geometry suggested some problems which neither was able to solve. As mathematicians explored these problems, calculus was born. Calculus had immediate application to the study of motion and curves, highly practical concerns of the day. It soon became the tool of the modern scientists, helping them to understand electricity and magnetism, for instance. A few steps later in the process came electric lights and radios.

Modern Mathematics 1800 AD → present

There are three aspects of modern mathematics which we will discuss briefly. The first is non-Euclidean geometry. By "non-Euclidean", I mean a way of doing geometry which leads to results that contradict the results Euclid got. For instance, in the non-Euclidean geometries we will discuss, there are no rectangles! Needless to say, this chapter may cause us to do some serious re-thinking.

Then we will discuss probability and statistics. While both these subjects had their roots several centuries earlier, they really came into their own as mathematical disciplines in the 19th and 20th centuries, and now greatly impact our daily lives. We will discuss some of the uses of probability in apologetics,

Not only will we discover that modern mathematics re-examined the foundations of geometry, we will also find an attempt to clarify some old issues about numbers and the concept of infinity. In the process, set theory was born. That subject will lead us to consider the subject of actual infinity, traditionally banned from mathematics until the turn of the 20th century.

This ends our very brief historical overview. It will be helpful if you keep these broad periods of history in mind as we discuss the various chapters. Appendix 1 contains a very brief historical timeline. It lists not only key names and topics in mathematics, but also some significant names and events from other aspects of history. It will be helpful for you to refer to it often.

Numeration Systems

Let go back in time and think a little about how numeration systems have developed throught the ages. There are four stages to look at.

Tally System

Since mankind has been sinful since the Fall, we have had to keep track of our possessions. This was true even before there were numbers. You've worked all day skinning animals and you've stacked the hides before retiring for the night. Are they all still there when you get up in the morning? How do you know if you can't count? One of the things that was done to take a tree limb or a bone and, with a stone, make a mark on it for each skin. So while you didn't know how many skins you had, you could make a 1-1 correspondence between the number off skins and the number of marks on the limb or bone. Instead of a limb of a tree, they often used the limb of an animal. Dead of course. This is an example of a tally system.

I once had a student who volunteered to tally the results of a class survey on the chalkboard. Her results looked something like the following:

Outcome A: | | | | | | |
B: | |
C: | | | | | | | | | |

The student had no concept of grouping. We've all been there. Think back far enough and you may be able to recall not wanting to trade five $1 bills for one $5.

Some very primitive cultures have a numeration system that consists of the numbers 1, 2, 3, many. So if there are 6 kids in the family, the parents would simply say there are many. They would, of course, recognize the difference between the "many" which represents 6 kids and the "many" that represents 4 kids.

Something to think about. In this system do we need a 0?

Simple Grouping

At some point in time, most cultures began to group in some way. Sometimes making groups of five and other times groups of ten. If we make groups of five, then our numerals may look something like the follow: |, | |, | | |, | | | |, V, V |, V | |, V | | |, V | | | |, VV, ... If we are grouping by fives, then every time we get five groups of something we must group them together. This requires a new symbol. For example, five V's could be written as N which would indicate one group of twenty-five. In this system, NNN V | | would represent 3 groups of 25, 1 group of 5 and 2 groups of 1 or 75 + 5 + 2 = 82 in our system. When we reach five groups of 25, then we would have to invent a new symbol to represent one group of 125, say Δ. So Δ Δ N VVV | | would represent 2(125) + 1(25) + 3(5) + 2 = 292.

At this stage in your life when you are able to group, would you now trade five $1 bills for one $5 bill? Of course you would.

Can symbols be arranged in any order in this system? Do we need a 0?

Multiplicative System

Somewhere along the line, someone got the bright idea that we could simply invent another symbol that would tell us how many of each group we have. So we could write NNNV | | as 3(N) 1(V)2(|). The Chinese and the Japanese both use three different number systems. In each case the most common system is a multiplicative one very similar to what we just described. Again, can we write this number in any order as long as the groupings remain the same? Do we need a 0?

Positioning System

In this system, an agreement is made that we will place the groups in a particular order. Usually, this means that we will start the groups of |'s on the right, move to the groups of 5, the groups of 25 and so on. Our number now becomes 3 | 5 which meant 3(25) + 1(5) + 2(1) = 82. This is a much more sophisticated system.

Order does matter, a 0 is needed.

Our system is positional, so 325 means that we have 3 groups of 100, 2 groups of 10 and 5 groups of 1's. Down the line when you become a parent, each of your children will go through each of these stages.

CHAPTER 2: A Historical Orientation **Homework**

1. Complete the following to find the product by the Egyptian method.

a. 5 x 6

1	6
2	12
4	24

b. 6 x 5

1	5
2	10
4	20

c. 9 x 7

1	7
2	14
4	28
8	56

d. 6 x 11

1	11
2	22
4	44

2. Use the Egyptian technique to find these products:

a. 5 x 7 b. 8 x 9 c. 12 x 8

3. Use the Egyptian technique to find these products:

a. 7 x 9 b. 5 x 8 c. 7 x 13 d. 11 x 6 e. 10 x 7

4. Use the Egyptian technique to find these products:

a. 15 x 13 b. 21 x 15 c. 19 x 22

5. Use the Egyptian technique to find these products:

   a. 14 x 25 b. 17 x 33 c. 22 x 13

6. Determine which of the following triples would belong in a Babylonian table of "Pythagorean triples":

   a. 14, 28, 30 b. 9, 16, 25 c. 11, 13, 17 d. 5, 12, 13

7. Determine which of the following triples would belong in a Babylonian table of "Pythagorean triples":

   a. 4, 5, 8 b. 7, 14, 15 c. 6, 8, 10 d. 8, 15, 17

8. A rectangular field is 400 feet long and 300 feet wide. You want to walk from one corner to the opposite corner. How many feet of walking would you save if you walk across the field instead of walking the length and width?

9. How high up a building will a 17 foot ladder reach if the base of the ladder is 8 feet from the base of the building?

10. Imagine that you had grown up in a culture that taught multiplication only by the Egyptian method. Then suppose in college you learned “our” method. Which method do you think you would prefer? Why?

11. Using the notation of the text, convert each of the following into a base 10 numeral.
    a. VVV||||
    b. NNV|||
    c. △△△ V|

12. Does the order of the symbols matter in our made-up numeration system? In other words can I write VVV||| as V|V||V?

13. Try to write 262 using our made-up numeration systems.

Selected Answers

1. a. 6+24 = 30 b. 10+20 = 30 c. 7+56 = 63 d. 22 + 44 = 66

2. a. 7 + 28 = 35 b. 72 c. 32 + 64 = 96

4. a. 13 + 26 + 52 + 104 = 195 b. 15 + 60 + 240 = 315 c. 22 + 44 + 352 = 418

6. a. no b. no c. no d. yes

7. a. no b. no c. yes d. yes

8. 200 feet

9. 15 feet (see 7 d)

11. c. 3 (5) + 4 = 19 b. 2(25) + 1(5) + 3 = 58 c. 3(125) + 5 + 1 = 381

12. No; the order does not matter.