CHAPTER 3: Logic

> Faith indeed tells what the senses do not tell, but not the contrary of what they see. It is above them and not contrary to them.
>
> The last proceeding of reason is to recognize that there is an infinity of things which are beyond it.[18]

Abstraction

The classical Greek period is the time in which some people would say the birth of mathematics occurred. Although there are hints in Babylonian civilization, it is only in Greece that abstract numbers clearly appear. Prior to this time, and in some cultures today, people might talk about two cows, two coins, or two days, but not "two" in and of itself. Counting might refer to "one finger, two fingers..." For the Greeks, "two" had an existence all its own, apart from any pairs of actual objects.

Likewise geometric abstraction was born in Greece. The word "geometry" means "earth measurement", which sounds rather concrete. But in Greece, lines in the sand or rectangles in a farmer's field were not the real lines and rectangles of geometry. Plato's ideals or ideal forms were the real "objects" of geometry, and they were very abstract.

There are several advantages of taking an abstract approach instead of always dealing with concrete or specific cases. Consider the following:

1. **Abstraction leads to a gain in generality.**
   For instance, the same rules apply to numbers whether you are working with sheep, coins, or days. So you can just learn the rules of arithmetic, and then (carefully) apply them in many diverse settings. Similarly, the area of a rectangle is found using the same formula whether the rectangle is a piece of cloth, a piece of wood, or a field.

2. **Abstraction frees the mind to concentrate on features of interest in a problem.**
   For instance, if you simply wanted to know how many sheep you had, you could ignore making any observations about their weight, color, or gender. For the purposes of this chapter, the analysis of reasoning will be similar: we will consider the abstract form of a logical argument apart from its actual content.

3. **Truth (in the Greek view) is about abstractions, so the study of mathematics is good (maybe even essential) preparation to study philosophy.**
   Concepts like beauty and justice were abstract to the Greeks. Learning to think about points and lines was easier.

Not only did the Greeks make use of abstraction in seeing the objects of mathematics (numbers and shapes) as abstract concepts, they also saw the method of mathematics as involving abstraction. This is particularly evident in their notion of proof based on deductive logic. Rather than analyzing an argument by looking at the details of its content, testing a deductive argument focuses on merely the form of the argument. In this way, the content (what the statements are about) is considered unimportant to the issue at hand, which is, is the reasoning correct?

How we know

The methodology of mathematics brings us to the general issue of methods of obtaining knowledge in any discipline or area of life. Obtaining knowledge is finding out what is true. Here are some of the ways people have suggested to do this:

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[18]Pascal, Pensees, p. 96.

1. Authority

To obtain knowledge from an authority is to quote someone, like Plato or Moses. Many people justify their claim to having the truth in just this way. However, the question of reliability is simply moved from you to your authority. This only establishes truth if the authority is accepted as a truthful source of knowledge.

2. Revelation

In a sense, this is just a special case of authority in which a person quotes God. The advantage is that once you have established that God is in fact the source of the revelation, that guarantees its truthfulness. The hitch is in getting everyone to accept first the existence of God and then the claim of revelation.

With respect to these two methods of obtaining truth, Morris Kline writes:

> We may pass over with a mere mention such sources of knowledge as authority and revelation, for these sources cannot be helpful in building mathematics or in acquiring knowledge of the physical world. It is true that in the medieval period of Western European culture men did contend that all desirable knowledge was revealed in the Bible. However in no significant period of scientific thought has this view played a role.[19]

At least with regard to revelation, we may not want to agree with Kline too quickly. I'm not sure Kline believes there is a God who reveals truth in Scripture. As Christians, we should allow for the possibility that the Bible has something to say about mathematics. In fact, there are Christians who believe that all the basic principles of mathematics have been revealed in the Bible, and that the rest of legitimate mathematics can be logically deduced from these principles. For instance, a brief article entitled "Math and the Bible" purports to present "the very beginnings of a Biblical construction for the foundations of arithmetic".[20] On the other hand, there are Christians who would dismiss this claim by asserting that God wrote the Bible to tell us about Himself, how to be saved, and how to live, but not how to do math. I'm not arguing for any one of these positions at the moment; I'm simply presenting some possibilities.

3. Experience and experimentation

Experience is a useful, but limited source of knowledge. You can learn that a fire is hot by putting your hand in it, but one would hope that there is a better way, like believing some authority. Experience based on careful observation can be very informative. While you personally may believe in the existence of the planet Mars because of an authority (you read about it in a book), the persons who first discovered Mars needed to be careful observers of the night sky.
Experimentation is planned and systematic experience. It increases the potential for learning immensely. The rise of modern science was due to a methodology which gave increased importance to experience and experimentation. They play a lesser role in mathematics.

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[19]Morris Kline, <u>Mathematics for the Nonmathematician</u>, p. 39.
[20]J. C. Keister, "Math and the Bible" in "The Trinity Review", No. 27, Sept./Oct., 1982, pp. 1-3.

4. Reasoning

There are three types of reasoning which we will mention.

a. Analogy

Reasoning by analogy argues that two situations have enough common features that what results in one situation will result in the other. For instance, you throw an apple into the air, and it falls down. By analogy, you reason that if you would throw a pear into the air, it would also fall down. Presumably, pears and apples are enough alike to make this analogy work.

On the other hand, you could argue this way. Spin a wet basketball on your finger, and the water will fly off into your face. By analogy, if the earth were spinning, the water in the oceans would fly off into space. Since the water in the oceans doesn't fly off, the earth must not be spinning. A long time ago, this analogy would probably have made perfect sense to people (except the part about the basketball). So why doesn't it work so well today? In any case, this points up a problem with analogies. You can't be sure that the reasoning works, because there may be differences which are really critical.

b. Induction

Induction builds on repeated occurrences of the same phenomena to guess that it will happen again. In science, this leads to "laws" like "what goes up must come down". For instance, if you watch enough apples fall from a tree, you become convinced that all the rest of the apples (and pears, for that matter) will fall with this type of reasoning. And the more apples you watch, the more convinced you become. The problem for Greeks doing mathematics was that the conclusion was never something you would know for sure. The Greeks considered mathematics to be about certain truth. And induction always leaves some room for doubt.

c. Deduction

Unlike arguments by analogy or induction, the conclusion of an argument by deduction is as <u>certain</u> as the information used in the argument. If you start with statements you already know to be true, deduction yields true conclusions. It is the process of deduction, then, on which we want to focus. Deduction does not provide us with a place to start, but it does provide a powerful tool to find more truth once we obtain a true foundation.

What we are planning to do is to test logical arguments for validity. Validity is not the same as truth. A statement is true or false, and to determine this one would need to know something about the meaning of the statement, and perhaps something about how that meaning compares to the real world. On the other hand, an argument is valid or invalid, and this depends on its form alone. The argument may contain false statements, and still be valid. Or, it may contain all true statements and be invalid. A valid argument contains good reasoning; an invalid argument contains bad reasoning.

Logical arguments

A logical argument consists of **premises** (or **hypotheses**) and a **conclusion**. In the context in which such arguments appear, the premises are accepted as true (at least for the sake of the argument). It is the intention of the argument to demonstrate or prove that the conclusion is true, not because of observing additional facts, but solely as a logical consequence of the premises. That is, in a valid argument, accepting the premises forces a reasonable person to also accept the conclusion.

A logical argument is **valid** if its conclusion follows necessarily from the premises. A logical argument is **invalid** if its conclusion is not necessarily implied by the premises.

All we are asking is whether the reasoning is correct. People working from false assumptions can reason quite correctly, and people working from true premises can reason quite incorrectly. IF the premises are true, and the deductive reasoning is valid, then the conclusion will be true. But at the moment, truth is not our focus; we are just looking at the reasoning part of the process.

Let's begin with an example.

Example 1:

Premises:	All cats have 4 legs.
	Sam is a cat.
Conclusion:	Therefore, Sam has 4 legs.

It is entirely possible that you know of a cat that does not have 4 legs. That is irrelevant at the moment. For the sake of the argument, for the purpose of testing its validity, we assume the premises are true. The issue is simply whether the information in the two premises (assumed true) is sufficient to force a reasonable person to believe the conclusion. Yes, it is.

Since content is not really the issue in testing arguments for validity, there are some decided advantages to looking for general patterns of valid arguments. One of the advantages of abstraction mentioned earlier was to avoid being confused by irrelevant features of the problem. In the example above, cats and legs and Sam are really irrelevant features when the issue at hand is testing the argument for validity. Let's be more abstract, and therefore more general, so that we could recognize this type of valid reasoning when the content of the statements is different. Let the letter "C" stand for cats, the letter "F" for four-legged things, and "s" for Sam (I used a small letter because Sam is an individual, whereas the capital letters stood for groups). Then the argument has this form:

Premises:	All C's are F's.
	s is a C.
Conclusion:	Therefore, s is an F.

What you see in this abstract argument is an example of what is called a **syllogism**. It is a form of reasoning, a pattern, which exemplifies one type of valid reasoning. The Greek philosopher Aristotle, who was a student of Plato (but who had some serious differences in his philosophy from that of Plato), was the first person known to have systematically studied syllogisms. He lived just before Euclid, and was the tutor of Alexander the Great, the conqueror of the Mediterranean world.

To use syllogisms to determine the validity of specific arguments requires that we have a list of syllogisms. Since syllogisms are merely "forms", the way to apply them is to try to find one that "fits" the argument you are testing. But what if none of the forms fits? Would that automatically mean that the argument was invalid? No, unless we knew somehow that our list of syllogisms included all the possible valid forms. Since I don't know that that will be true, we will have another approach to deciding that an argument is not valid. We will also have a list of typical "invalid" forms for arguments. The arguments we will consider in this class will generally match a form in one list or the other.

Here's a list of syllogisms for valid arguments, with names so that we can refer to them easily. This list is by no means exhaustive, but it's a start. (Note that the DIRECT syllogism is the form in the earlier example.)

SYLLOGISMS

DIRECT	
Premises:	All X's are Y's.
	z is an X.
Conclusion:	Therefore, z is a Y.
INDIRECT	
Premises:	All X's are Y's.
	z is not a Y.
Conclusion:	Therefore, z is not an X.
TRANSITIVE	
Premises:	All X's are Y's.
	All Y's are Z's.
Conclusion:	Therefore, all X's are Z's.

Example 1 (about Sam the cat) is a DIRECT argument; it is valid because it fits the form the DIRECT syllogism. Here are two examples of the other two types of syllogisms. Can you tell which is which?

Example 2:

Premises:	All Germans are rational people.
	All rational people are boring.
Conclusion:	Therefore, All Germans are boring.

Example 3:

Premises:	All professors are old.
	Tommy is not old.
Conclusion:	Therefore, Tommy is not a professor.

If you recognized Example 2 as a TRANSITIVE syllogism, and Example 3 as an INDIRECT syllogism, you were correct. Remember, the question is not whether the statements (like the premise, "All professors are old") are true, but whether the reasoning is valid. In each example above, if you accept the premises as true, then you are logically forced to accept the conclusions as true.

Now it is time to consider some **invalid** arguments. Again, let's start with an example.

Example 4:

Premises:	All cats have 4 legs.
	Sam has 4 legs.
Conclusion:	Therefore, Sam is a cat.

Now, if this is the same Sam as in Example 1, then the conclusion is true. But, that really doesn't matter at the moment. Our concern is whether this argument is valid, whether the conclusion follows logically from these two premises. Simply knowing that Sam has 4 legs does not force us to conclude that Sam is a cat; she ("Sam" in this case is short for Samantha) might be a dog or a horse or a chair (OK, maybe you never named a chair, but some people might!).

Let's take a look at another example of the same type.

Example 5:

Premises: All the books in the Bible are inspired.
The Book of Psalms is inspired.
Conclusion: Therefore, the Book of Psalms is in the Bible.

This example could be quite tricky. If you are thinking, "Well, of course Psalms is in the Bible! That's obviously valid!", you're correct about Psalms being in the Bible, but wrong about the argument being valid. Remember, validity is not about the truth of the conclusion, but whether the conclusion follows by logical reasoning from the premises. If all you knew was what the premises told you (try hard for the moment to ignore everything else you know), would you be forced by the two premises to believe that Psalms is in the Bible? No, not really. For instance, the first premise allows for the possibility that there are inspired books which are not in the Bible. (Remember, for the sake of this argument, you can't bring in other information you know.) Since that is a possibility, Psalms might be one of them. A person who reasons like Example 4. may not need to change his theology, but needs some correction in logic.

Let me say that last sentence in a slightly different way. Showing that a person has reasoned incorrectly does not necessarily mean that you have shown the person's conclusion to be false. The conclusion might be true, but a different argument is needed to correctly demonstrate that fact. Paul tells in Ephesians that we should speak the truth in love. I would simply add that I believe we should speak the truth with correct reasoning.

Perhaps another argument in the same form will help. In this case, I suspect you may not want to be quick to endorse the conclusion, even if you agree with the premises.

Example 6:

Premises: All men over 18 have the right to vote.
Chris has the right to vote.
Conclusion: Therefore, Chris is a man over 18.

Chris might be a woman! The argument is invalid.

Invalid arguments which have standard forms are called **fallacies**. We will consider three fallacies which in a sense correspond to the syllogisms above. (Again, this list is far from exhaustive.) What you will notice is that they look deceptively similar to the syllogisms. If you look carefully at the abstract forms, they are clearly distinguishable. In everyday conversations, or in textbooks, or political or religious debates, they may not always be as easy to spot. First, let's list the abstract forms, again with names so that we can refer to them.

FALLACIES

CONVERSE	
Premises:	All X's are Y's.
	z is a Y.
Conclusion:	Therefore, z is an X.
INVERSE	
Premises:	All X's are Y's.
	z is not an X.
Conclusion:	Therefore, z is not a Y.
COINCIDENCE	
Premises:	All X's are Y's.
	All X's are Z's.
Conclusion:	Therefore, all Y's are Z's.

Examples 4 , 5 and 6 are Converse fallacies. Here's an example of a Coincidence fallacy.

Example 7:

Premises:	All Fundamentalists are Conservatives.
	All Fundamentalists are Creationists.
Conclusion:	Therefore, all Conservatives are Creationists.

The above approach to analyzing arguments for validity depends on having the forms of the syllogisms and fallacies, and the ability to decide which form fits the argument at hand. A less formal, but more visual/geometric approach may be useful at times, or simply more appealing to some readers.

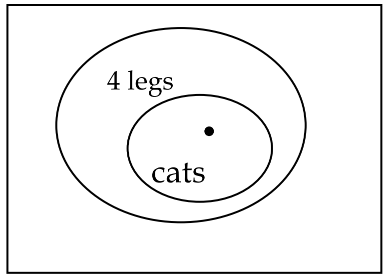
A few general guidelines will be given for the construction of a "picture" of the argument. If the only possible ways the picture can be drawn force the conclusion of the argument to be true, then the argument is valid. If, on the other hand, there is a way to draw the picture for which the conclusion is not true, then the argument is invalid.

Here are the guidelines:

1. Groups such as "all cats" are represented by circles meant to include all members of the group inside them.
2. Individuals are represented by dots.
3. "All X's are Y's" is represented by one circle (the X's) completely surrounded by a second circle (the Y's).
4. "No X's are Y's" is represented by two circles that don't overlap.

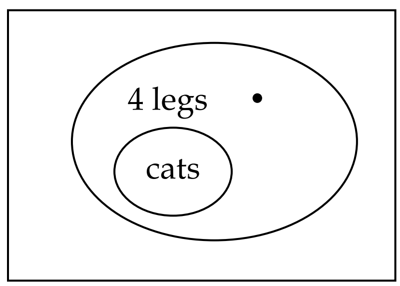
Figure 1 below returns us to Example 1 of our earlier discussion. The smaller circle represents "all cats", while the larger circle represents all things having "4 legs". The dot represents Sam. This is the only possible way to draw this picture according to the guidelines. Clearly, the dot is in the larger circle, i.e., we are forced by the premises to conclude that Sam has 4 legs. The argument is seen to be valid.

FIGURE 1



As an example of an invalid argument, consider Example 4 again. A possible way (not the only way) to draw the picture for this argument is shown in Figure 2 below. In this case, the dot representing Sam is not in the small circle, indicating Sam is not a cat. Since this is possible, the conclusion is not necessarily true, and so the argument is invalid.

FIGURE 2



The picture could also be drawn just like it was in Figure 1. From this we conclude that saying an argument is invalid is NOT the same as saying that the conclusion is false. Sam might indeed be a cat. Our argument in this example is simply inadequate logically to prove that conclusion.

As a final type of argument, we wish to consider the use of statements like "No X's are Y's." Guideline 4 indicates that the picture of such a statement will involve two circles which do not overlap. Here's an example.

Example 8:

Premises: No unbelievers will be saved.
All of the elect will be saved.
Conclusion: Therefore, none of the elect will be unbelievers.

By the way, a statement of the form "No X's are Y's" could be changed so that we could try to use syllogisms and fallacies. We would need to conceive of a new group, the "not-Y's". The statement then becomes, "All X's are not-Y's". For example, "No cats are dogs" becomes "All cats are not-dogs". In a picture, if the Y's are represented by a circle, then the not-Y's would be represented by everything outside the circle. Unfortunately, such a transformation would be of little help in this example; the resulting form doesn't match any of our 6 forms.

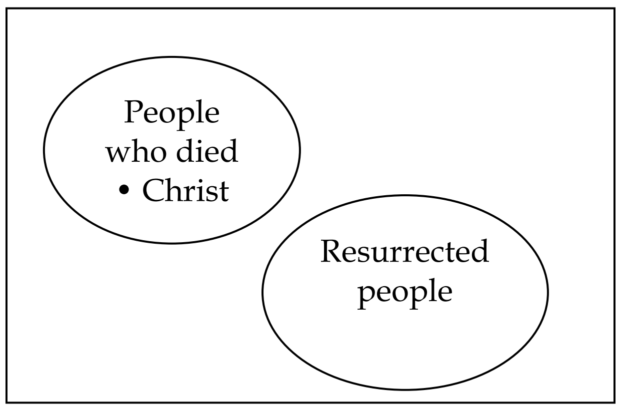
As a final example, consider the following argument presented by the Apostle Paul in 1 Corinthians 15: 13, "If there is no resurrection of the dead, then not even Christ has been raised." Paul has not stated the argument in the style which we have been using, so let me restate it:

Premises: All people who die do not rise from the dead.
Christ is a person who died.
Conclusion: Therefore, Christ did not rise from the dead.

If you let X represent "persons who die" and Y represent "things that do not rise from the dead", then this fits the Direct syllogism form, using a little freedom with the English language to make the sentences sound better. If you phrase things a bit differently, the picture approach is quite simple:

Premises: No people who die are resurrected.
Christ is a person who died.
Conclusion: Therefore, Christ was not resurrected.

FIGURE 3



This argument is a perfect example of a situation in which the argument is part of a larger argument. Paul in fact has argued that Christ WAS raised from the dead! He reasoned along several lines. 1 Corinthians 15 opens with a statement that Christ's resurrection was "according to the Scriptures" (v. 4) - a use of revelation. He then cites some authorities - Peter and the Twelve, as well as over 500 brothers (v. 5-6). Then there is his own personal experience (v. 8). With the resurrection of Christ established in these ways, Paul moves on to the issue of a general resurrection of the dead. Apparently Paul had heard that someone in Corinth was saying that there was no resurrection of the dead. In his argument to show that this teaching was false, Paul incorporated the argument we examined above. Just as an aside, this example shows how important considering the context of a verse can be to obtaining a correct understanding of it.

Compound statements.

All of the above arguments involve simple statements. But what do we do if the argument involves compound statements; that is, statements that combine simple statements using the connectors, *and*, *or*, or *If..., then. ...* (While there are other connectors, we will limit our discussion to these three.)

1. The word "*and*" placed between two statements forms a new statement called a **conjunction.**

Example: Statement #1: I went to church.
Statement #2: I went to lunch.
Conjunction: I went to church *and* I went to lunch.

In general, if p and q are any two statements, then " *p and q*" is the conjunction of p and q. We often write " *p and q*" as " $p \wedge q$" where $\wedge$ represents *and*. In order for "$p \wedge q$" to be true, both p and q must be true. In order for the above example to be a true statement, I must go both to church and to lunch. If I fail to go to one or the other or both, the statement is false. These ideas are summarized in the following truth table.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

2. The word "*or*" placed between two statements forms a new statement called a **disjunction**. Using the same statements as above, "Iwent to church *or* I went to lunch" is an example of a disjunction. Using symbols, we write "*p or q*" as "$p \vee q$." If this disjunction is to be a true statement, then I must go to church or to lunch or to both. The only time the statement is not true is when I fail to go to both places. Again we can summarize this idea using a table.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

3. Most of the theorems that you are familiar with from your study of mathematicsare in the form of "*If p, then q*". This type of compound statement is called an **implication** or a **conditional**. In our example above, our implication would be, "If I went to church, then I went to lunch." In symbols, we write any "If p, then q" statement as $p \rightarrow q$. Before we look at the truth table let's look at an example.

At some time or other we have all heard someone make the statement that their Dad said
If you get all A's on your report card, then I will buy you a car.

Essentially, our implication is: If you get all A's , then I'll buy you a car.

Let's consider the possibilities.
a. The student gets all A's and the dad buys the car. In this case the implication would be true. The Dad kept his promise.
b. The student gets all A's and the dad does not buy the car in which case the implication would be false. The Dad did not keep his promis.
c. The student does not get all A's which relieves the dad of any obligation and he is free to either purchase a car or not. His promise is not broken in either case and, therefore, the implication is true.

While we cannot prove the entries in the table below (they must be accepted as true without proof), we can recite enough examples as the one above to at least give you a feeling that you would want the implication to be false only in the case where a *true* statement implies a *false* statement. The truth table for an implication is as follows:

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Now that we have looked at these compound statements and determined under what conditions we would want them to be true, we are now ready to look at arguments involving compound statements. We will keep the arguments "simple" in that we will use only two premises.

Example #1 If John gets up on time, then John will go to class.
John gets up on time.
John goes to class.

Solution: The first thing we must do is to translate the argument into symbols.
If we let p represent "John gets up on time" and q represent "John will go to class", then we can write the argument as

$$p \to q$$
$$\underline{p\quad}$$
$$q$$

The next step involves writing the argument in horizontal form:

$$[(p \to q) \wedge p] \to q$$

This looks very much like an algebraic expression that we would have studied in an algebra course and we will work with it in much the same way. We will start by working within the parentheses to determine the truth value of $p \to q$, add to that the truth value of p and have all of that impy q.

We then construct a table using the statements $p,\ q,\ p \to q,\ (p \to q) \wedge p,$ and $[(p \to q) \wedge p] \to q$ as our column headings and complete the table using the rules for conjunctions and implications.

p	q	$p \to q$	$(p \to q) \wedge p$	$[(p \to q) \wedge p] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

If the entries in the last column are all T's, then the argument is a valid one; that is, the conclusion is forced to follow from the list of premises. This is a valid argument.

Example #2: If John studies, then he passes the exam.
John passes the exam.
John studies.

Following the same procedures, we get

(a) $p \to q$
q
p and (b) $[(p \to q) \wedge q] \to p$

Constructing and completing the table yields

p	q	$p \to q$	$(p \to q) \wedge q$	$[(p \to q) \wedge q] \to p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

Since we have an entry in the last column that is *false*, the argument is invalid. You probably recognize this as the *fallacy of the converse.*

Some years ago, I, Dave, was involved along with a number of other people in a study to determine if women should be ordained as elders. Our moderator gave the following argument as justification for the ordination of women.

If a person is an elder, then the person is a servant. The best servants in any church are women. Therefore, woman ought to be elders.

Regardless of which side of the issue you are on, this is not a valid argument; in fact, it is the same argument that we looked at in Example #2. When challenged, the moderator acknowledged that it was not a good argument and continued on his way. There are people who will make an attempt to use an invalid argument to make their point especially in emotionally charged cases where the presenter assumes the audience sees the conclusion as a true statement. Does it matter to you how one arrives at a conclusion?

An interesting argument that appeared in a "Dennis the Menace" cartoon several years ago boiled down to this:

If I send a letter to someone who does not exist, then the letter is returned to me.
The letter was not returned to me.
The letter was sent to someone who does exist.

If p represents "I send a letter to someone who does not exist.", then ~p represents the negation of p and would represent the statement "The letter was set to someone who does exist."

The argument translates to:

$$\begin{array}{l} p \to q \\ \underline{\sim q\quad} \\ \sim p \end{array} \qquad \text{and} \qquad [(p \to q) \wedge \sim q] \to \sim p$$

The table looks like this:

p	q	$p \to q$	$\sim q$	$(p \to q) \wedge \sim q$	$\sim p$	$[(p \to q) \wedge \sim q] \to \sim p$
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

and the argument is a valid one. That means that if I send a letter to Santa Claus and it is not returned to me, then it must have been sent to someone who exists; eg, Santa Claus. We will leave to you to decide whether the conclusion is a true statement or not.

CHAPTER 3: Logic **Homework**

Determine which arguments are valid and which are invalid. Identify the syllogism or fallacy, if possible.

1.	Premises:	All Germans are rational.
		Hans is rational.
	Conclusion:	Therefore, Hans is German.
2.	Premises:	All roses are red.
		Pete is a rose.
	Conclusion	Therefore, Pete is red.
3.	Premises:	All artists are creative.
		All musicians are creative.
	Conclusion:	Therefore, some musicians are artists.
4.	Premises:	No TV shows are worth watching.
		"60 Minutes" is a TV show.
	Conclusion:	Therefore, "60 Minutes" is not worth watching.
5.	Premises:	All human beings are sinners.
		Michael is not a sinner.
	Conclusion:	Therefore, Michael is not a human being.
6.	Premises:	All roses are red.
		Elmo is not a rose.
	Conclusion:	Therefore, Elmo is not red.
7.	Premises:	All politicians are rich.
		No Democrats are rich.
	Conclusion:	Therefore, no Democrats are politicians.
8.	Premises:	All dogs are mammals.
		All mammals are animals.
	Conclusion:	Therefore, all dogs are animals.
9.	Premises:	All dogs are cats.
		All cats are birds.
	Conclusion:	Therefore, all dogs are birds.
10.	Premises:	No cats are rats.
		No rats are dogs.
	Conclusion:	Therefore, no cats are dogs.
11.	Premises:	No classes are easy.
		The Nature of Math is a class.
	Conclusion:	Therefore, The Nature of Math is easy.
12.	Premises:	All athletes are healthy.
		Jim is healthy.
	Conclusion:	Therefore, Jim is an athlete.

13. Premises: All spiders are scary.
Godzilla is not a spider.
Conclusion: Therefore, Godzilla is not scary.

14. Premises: All friends are loyal.
Judas was not loyal.
Conclusion: Therefore, Judas was not a friend.

15. Premises: All rectangles are squares.
All squares are triangles
Conclusion: Therefore, all rectangles are triangles.

16. Premises: All children are silly.
No professors are children.
Conclusion: Therefore, no professors are silly.

17. Premises: All men are sinners.
Peter is a man.
Conclusion: Therefore, Peter is a sinner.

18. Premises: No Christians are perfect.
Helen is a Christian.
Conclusion: Therefore, Helen is not perfect.

19. Premises: No rich men will enter heaven.
All humble people will enter heaven.
Conclusion: Therefore, no rich men are humble.

20. For each of the following, construct a truth table to determine its valdity. If you recognize that an argument has the same format as a previous one, there is no need to construct the table again. However, you must state whether the argument is valid or invalid.

a. If Fred is a Christian, then he is a servant.
Fred is a Christian.
Fred is a servant.

b. If the weather is bad, then the game is cancelled.
The game is cancelled.
The weather is bad.

c. If I earn enough money, then I am able to stay in school.
I am able to stay in school.
I earned enough money.

d. If I go to Campbell's, then I'll bring you back a soda.
I bring you back a soda.
I went to Campbell's.

e. If I am a student, then I do my homework.
<u>I am a student.</u>
I do my homework.

f. If I go and prepare a room for my guest, then I will return for him.
<u>I return for him.</u>
I prepared a room for him.

21. In each of the above exercises, the number of combinations of truth values for *p* and *q* is four: TT, TF, FT, and FF. How many combinations would we have if we had three statements, say *p*, *q* and *r*? What if we had four statements? Five statements? **n** statements?

Selected Answers: Chapter 3

1. invalid, converse fallacy
2. valid, direct syllogism
3. invalid
4. valid
5. valid, indirect syllogism
6. invalid, inverse fallacy
7. valid
8. valid, transitive syllogism
9. valid, transitive syllogism
10. invalid
11. invalid
12. invalid, converse fallacy

20. a. Valid b. Invalid c. Invalid