CHAPTER 12 : Chinese Mathematics

> If we do not know ourselves to be full of pride, ambition, lust, weakness, misery, and injustice, we are indeed blind.[31]

Relatively speaking, we have little information about the development of mathematics in Chinese civilization. However, there is enough information to provide a basis for some general observations about Chinese mathematics, as well as to illustrate some specific types of problems which were addressed and solved. One difficulty encountered in attempting to trace the chronological development is the dating of specific works. The materials used for written records were not as long-lasting as the tablets of Babylon or the papyrus of Egyptian deserts. Some dynasties opposed learning, and destroyed books, as happened around 213 BC. The result is that we know of Chinese mathematicians and their works from copies dated centuries after the originals were written and without a clear sense of the transmission of the text. Nonetheless, we have sufficient information to conclude that early Chinese civilizations made significant discoveries, many of which predated the same discoveries in Greek or European culture.

Early Chinese arithmetic is positional, but with no zero. The base is 10 so it strongly resembles our system. Early aids to calculation were counting rods; numerals used line segments and are called rod numerals. This is somewhat similar to the numerals of the Mayans. The abacus, or counting board, often associated with the Oriental cultures, was actually introduced at a relatively late date, perhaps about 1100 AD. (Similar instruments were used in Roman times in Europe.) But once introduced, the Chinese made extensive and highly effective use of the abacus.

One of the earliest Chinese mathematics books is called Arithmetic Classic of the Gnomon and the Circular Paths of Heaven. It contains a diagram "proof" of Pythagorean theorem, dating from possibly 600 BC. That would put it about a century before Pythagoras.

The Nine Chapters on the Mathematical Art is the oldest textbook on arithmetic and its applications in existence, possibly dating from around 100 B.C. It was used as a textbook in China for over a millennium; this compares to Euclid's Elements, which was used as a textbook on geometry in western Europe for almost two millennia. The Nine Chapters is a practical handbook of problems. Diagrams serve as a guide to the algebraic solutions of problems. Procedures are explained only by examples; there are no "formulas" or notation which you might associate with algebra. Like a modern-day textbook, some of the problems seem artificial: in real life, a person would never use algebra to find the answer to the question.

Here's an example: "There are 9 equal pieces of gold and 11 equal pieces of silver. The two lots weigh the same. If one piece is removed from each lot and put in the other, the lot containing mainly gold is found to weigh 13 ounces less than the lot containing mainly silver. Find the weight of each piece of gold and silver."

The algebraic methods used to solve the problems in the Nine Chapters are quite advanced; some continued to be used in western Europe until 1500 AD. A significant first is that the Nine Chapters contains the first known use of negative numbers in problem-solving, although negatives are not used as answers, meaning that they were probably not given full number status. The Nine Chapters also contains some geometry; as with many ancient cultures, some of the geometric "formulas" turn out to be incorrect.

The Sea Island Mathematical Manual was a collection of nine problems with solutions and commentary by a scholar named Liu Hui, who lived around the third century A.D. The name of this book is derived from the first problem: how to find the height and distance of an island. The solution presented makes elaborate use of similar triangles which resembles but does not really achieve the approach through trigonometry which would be more natural today.

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[31]Pascal, Pensees, p. 152.

About the same time Sun Tsu wrote Master Sun's Mathematical Manual. The most famous problem contained in this work is this one: "We have things of which we do not know the number; if we count by threes, the remainder is 2; if we count by five's, the remainder is 3; if we count them by sevens, the remainder is 2." The answer is given as 23, which is correct since $23 = 3 \bullet 7 + 2$, $23 = 4 \bullet 5 + 3$. and $23 = 7 \bullet 3 + 2$. A solution is proposed which generalizes to some extent. A general solution (that is, a description of all the numbers which satisfy the given conditions) was given by the Chinese in the 12th century. This general solution to the problem is now known as the Chinese Remainder Theorem.

The homework at the end of this section requires you to solve simpler problems of this sort. To make the problem relate more naturally to what we have done, let me rephrase it this way: Find the number in base 10 which in base 3 ends in a 2 , in base 5 ends in a 3, and in base 7 ends in a 2. You see, 23 is the same number as 212_3 and 43_5 and 32_7. The Chinese developed a fairly sophisticated way to solve such problems. We will solve only simple problems, and use the following method.

Example: Find a number in base 10 which in base 5 would end in 1 and in base 6 would end in 2.

> We will do our work in base 10. The numbers in base 5 which end in 1 can be listed in base 10 by starting with 1 and adding 5's: 1, 6, 11, 16, 21, 26, 31,..... The numbers in base 6 which end in 2 can be listed by starting with 2 and adding 6's: 2, 8, 14, 20 26, 32,..... What we are looking for is a number which appears in both lists. In fact, if we work on both lists together, there is no need to go past the first such number. In this case, notice that 26 is in both lists, and hence solves our problem. To be very specific, notice that 26 can be written as 101_5 and 42_6.

Example: Find a number in base 10 which in base 4 would end in 3 and in base 6 would end in 2.

> If you write the lists of numbers for this problem as we did above, you should get 3, 7, 11, 15, 19, 23, 27, 31,.... and 2, 8, 14, 20 26, 32,.... So far, no matches. If you look carefully, you should observe that the numbers in the first list are all odd, and the numbers in the second list are all even. A few moments reflection will probably enable you to prove that there is no number that solves this problem: adding an even number to an odd number always results in an odd number, and adding an even number to an even number always results in an even number. We proved these statements in Chapter 8.

Moving to a completely different problem, a significant result was achieved by the astronomer and mathematician Tsu Ch'ung-chih (430-501 AD) when he rationally approximated π as 355/113. This approximation is correct to 6 decimal places. As far as we know, this was the best value in the whole world for the next 1000 years.

The Chinese developed the first known table of the trigonometry function "tangent" in the eighth century AD. As with later trigonometric concepts, it was used for purposes related to astronomy and the calendar. Apparently no further trigonometry was developed in China for 1000 years.

We have seen how the Pythagorean belief that "everything is number" led them to reject irrational numbers, and how the Hindus developed the concept of 0 and the Mayan interest in calendars influenced their numeral systems. How was the development of Chinese mathematics related to the development of Chinese culture generally? As with many early civilizations, there was a practical orientation: mathematics was needed to develop the calendar and to assist in governmental administration. Relatively early, the Chinese developed mathematics sufficient for these purposes. Why then did mathematics not develop further in China? The following are two suggestions.

One historian has suggested that "to a large extent progress in mathematics was stifled by the general Chinese reverence for the past" [32] "Stifled" suggests a judgment about values. I would suggest that

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[32]Victor J. Katz, A History of Mathematics, HarperColllins, New York, 1992, p. 198.

if the connection between reverence for the past and little progress in mathematics is a valid one, we should not be too quick to criticize those who would chose reverence of the past.

Another suggestion is connected to Morris Kline's claim that the story of mathematics is inherently connected to the story of science. Modern science, along with mathematics, developed in western Europe in a way that did not occur in China. Why? "The predicament of China, both ancient and modern, is an eloquent though tragic witness to the need of natural theology if science is to flourish. The implicit denial of natural theology in ancient China reached its highest level in the logic-defying aphorisms of Taoism..."[33] Here the claim is being made that there is a religious reason why science, and hence mathematics, did not develop further in China as it did in Europe. Here's a question to ponder: Is Christianity really a significant part of the cultural environment in which mathematics has undergone significant development? If so, what exactly is its contribution to mathematics?

CHAPTER 12: Chinese Mathematics **Homework**

1. Find a number in base 10 which in base 5 would end in 2 and in base 6 would end in 3.

2. Find a number in base 10 which in base 4 would end in 2 and in base 7 would end in 4.

3. Find a number in base 10 which in base 6 would end in 1 and in base 7 would end in 3.

4. Find a number in base 10 which in base 6 would end in 2 and in base 4 would end in 3.

5. Find a number in base 10 which in base 3 would end in 1, in base 4 would end in 2 and in base 5 would end in 2.

6. Find a number in base 10 which in base 3 would end in 2, in base 4 would end in 3 and in base 5 would end in 1.

Selected Answers:

1. 27 3. 31 4. does not exist

[33] Stanley L. Jaki, The Road of Science and the Ways to God, University of Chicago, Chicago, 1978, pp. 14-15.