

Nature of Math
Test 2

Study Guide

About the Test

- 34 Questions
- 20% of Semester Grade
- Calculator Okay
- 75 minutes
- Sample Questions in Appendix



Suggestions

- Look through sample questions
- Get enough sleep
- Visit Math Lab
- Study with friends

Topics

- Averages & Weighted Averages
- Counting Combinations
- Algebra: Multiplying / Combining Like Terms
- Different Bases
 - Whole Number base conversions
 - ~~Decimal / Fraction conversions~~ (Not on test)
 - Mayan Numbers – Normal & Solar Numbers
 - Unknown Bases / Finding Numbers
- Using Exponents

Averages

- Ordinary Average
 - Add the numbers
 - Divide by the number of numbers

$$\frac{85 + 90 + 93}{3} = 89.3$$

Averages

- Weighted Average
 - Multiply each number by its weight
 - Add
 - Divide by the total weight

Calculate your GPA if you get:

$$\begin{array}{l}
 \text{a B in a 4-unit class} \quad B + A + B + C \\
 \text{an A in a 2-unit class} \quad (4)3 + (2)4 + (3)3 + (3)2 \\
 \text{a B in a 3-unit class} \quad 12 \\
 \text{a C in a 3-unit class} \quad \frac{12+8+9+6}{12} = \frac{35}{12} = 2.92
 \end{array}$$

Averages

- Average Speed
 - Determine distance for each part
 - Total Distance / Total Time

Find your average speed if you ride your bike

$$\begin{array}{l}
 \underbrace{20 \text{ mph for } 30 \text{ minutes}}_{10 \text{ miles}} \text{ then } \underbrace{15 \text{ mph for } 1 \text{ hour}}_{15 \text{ miles}} \\
 \text{avg. speed} = \frac{\text{total miles}}{\text{total hours}} = \frac{10+15}{1.5} = \frac{25}{1.5} = 16.67 \text{ mph} = 16 \frac{2}{3} \text{ mph}
 \end{array}$$

Counting Combinations

- "Basic Counting Principle"
 - Count ways things can be done
 - Multiply

If you can order three different soups and four different pastas for a dinner combo, how many total choices do you have?

$$3 \times 4 = 12$$

Bases – Whole Numbers

- Converting from Base 7 to Base 10
 - Determine Place values for each place
 - Add up the digits

$$\begin{array}{r} 1 \ 3 \ 4 \\ \hline 49 \ 7 \ 1 \end{array} \quad 7$$

$$1 \times 49 + 3 \times 7 + 4 \times 1$$

$$49 + 21 + 4$$

$$74$$

Bases – Whole Numbers

- Converting to a different Base
 - Determine place values then guess
 - Repeated long division

Write 32 in base 7

$$\begin{array}{r} 4 \ 4 \\ \hline 7 \ 1 \end{array} = 44_7$$

Write 500 in base 7

$$\begin{array}{r} 71 \\ 7 \overline{)500} \\ \underline{49 \times} \\ 10 \\ \underline{7} \\ 3 \end{array} \quad \begin{array}{r} 10 \\ 7 \overline{)71} \\ \underline{7 \times} \\ 10 \\ \underline{7} \\ 3 \end{array} \quad \begin{array}{r} 1 \\ 7 \overline{)10} \\ \underline{7} \\ 3 \end{array}$$

1313₇

Bases – Arithmetic

- Addition
- Multiplication

$$\begin{array}{r} 1 \\ 45_7 \\ + 34_7 \\ \hline 112_7 \end{array}$$

Bases – Unknown

- Number written in unknown base
- Unknown number (Chinese Remainder Theorem)

54 is written as 42_x where x is an unknown base – find the base

$$\begin{array}{r} 42 \\ \hline \times 1 \end{array}$$

$$4x + 2 \cdot 1 = 54$$

$$4x = 54 - 2$$

$$4x = 52$$

$$x = 13$$

Bases – Unknown

- Number written in unknown base
- Unknown number (Chinese Remainder Theorem)

A number which in base 5 ends in 3 and in base 7 ends in 4 is: _____

$$3, 8, 13, 18, 23, 28, 33, 38, 43$$

$$4, 11, 18, 25, 32$$

$$18$$

$$33_5 = 24_7$$

Mayan Numbers

- Base 20
- Digits written as: 5 = | and 1 = .
- "Common" Mayan: 400 / 20 / 1
- "Solar" Mayan: 360 / 20 / 1

Add: $\begin{array}{r} | | | ; \dots \\ + | | | ; | \dots \\ \hline \end{array}$

Add:

$$\begin{array}{r} \text{|||} ; \dots \\ + \text{|||} ; | \dots \\ \hline \text{|||||} ; | \dots \end{array}$$

Add:

$$\begin{array}{r} \text{|||} ; \dots \\ + \text{|||} ; | \dots \\ \hline \text{|||||} ; || \dots \end{array}$$

Add:

$$\begin{array}{r} \text{|||} ; \dots \\ + \text{|||} ; | \dots \\ \hline \cdot ; || \quad | \quad \cdot \end{array}$$

Add:

$$\begin{array}{r} \text{|||} ; \dots \\ + \text{|||} ; | \dots \\ \hline \cdot ; || \quad | \quad \cdot \\ \hline \cdot ; || ; || \cdot \\ \hline \end{array}$$

Add:

$$\begin{array}{r} \text{|||} ; \dots \\ + \text{|||} ; | \dots \\ \hline \cdot ; || \quad | \quad \cdot \\ \hline \cdot ; || ; || \cdot \\ \hline \end{array}$$

400 20 1

$$1 \times 400 + 10 \times 20 + 11 \times 1$$

$$400 + 200 + 11$$

$$611$$

Exponents

Rules For Exponents

1. $a^m \cdot a^n = a^{m+n}$ (m+n is the total number of a's)
2. $\frac{a^m}{a^n} = a^{m-n}$ (cancel n of the m a's)
3. $(a^m)^n = a^{m \cdot n}$ (n groups of m a's each)
4. $(a \cdot b)^m = a^m \cdot b^m$ (regroup the a's and b's)

Exponents

1. Zero as an exponent

$$a^0 = 1$$

since $1 = \left(\frac{a^n}{a^n}\right) = a^{n-n} = a^0$.

Examples: $7^0 = 1$ $256^0 = 1$

2. Negative whole numbers as exponents

$$a^{-n} = \frac{1}{a^n}$$

since $a^n \cdot a^{-n} = a^{n-n} = a^0 = 1$.

Examples: $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ $4^{-1} = \frac{1}{4}$ $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

$$x^7 \cdot x^{10} = x^{17}$$

$$(x^7)^{10} = x^{7 \cdot 10} = x^{70}$$

$$\frac{x^{20}}{x^5} = x^{15}$$

$$4^0 = 1$$

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$16^{0.5} = \sqrt{16} = 4$$

$$9^{1.5} = 9 \cdot 9^{0.5} = 9 \cdot \sqrt{9} = 9 \cdot 3 = 27$$

$$x^7 \cdot x^{10} = x^{17}$$

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Exponents – Larger Exponents

- Addition $8+10$
- Multiplication 8×10
- Exponents 8^{10}
- Arrow Notation $8 \uparrow \uparrow 10$
- Arrow Notation $8 \uparrow \uparrow \uparrow 10$

(This list goes from smaller to larger)