CHAPTER 11: Native American Mathematics

So I hold out my arms to my Redeemer, who having been foretold for four thousand years, has come to suffer and to die for me on earth, at the time and under all the circumstances foretold. By his grace, I await death in peace, in the hope of being eternally united to Him. Yet I live with joy, whether in the prosperity which it pleases Him to bestow upon me, or in the adversity which He sends for my good, and which He has taught me to bear by His example.30

Maya (1000 BC on; 200-1000 A.D. Classical Period)

Mathematics in Mayan culture was developed for use in architecture, as well as in astrology/astronomy. In the latter connection, the Mayan calendars are of particular note. The Mayans also had a developed form of writing, which aids our understanding of their mathematics.

The common Mayan numeration was a combination of grouping by five's and a positional base twenty system. Only three symbols were used: a position-holder for zero, a symbol resembling a half-closed eye ".timezone"; a single dot "." for one; and a bar "|" for five. Thus ".\ldots." would represent 4, and ".|." would represent $5 + 1 = 6$. Hence up to three bars and four dots could appear in a single position. The simple explanation for these symbols is that dots are fingers; when you get to five fingers, you have one hand symbolized by a line. Once you reach four hands (actually, two hands and two feet), it's necessary to "carry" one group of twenty (equal to 1 person), so that you can start using your fingers and hands again. For instance, 24 in Mayan would look like ".|;\ldots;.;", where I have used the semi-colon to separate the one's column from the twenty's column. 24 (2 groups of 10 and 4 left over) has been regrouped into 1 group of 20 and 4 left over. The Mayans wrote their numbers vertically, but the semicolon with symbols written horizontally is more convenient for us.

Here are more examples from Mayan to base 10:

Example 1: 

\[
\begin{align*}
|| ; || & = (10) (20) + (5+5+5+1) = 216 \\
||| \ldots ; . & = (19) (20) + 2 = 382 \\
|.. ; \equiv & = (7)(20) + 0 = 140 \\
\ldots ; |; & = (2)(20\cdot 20) + (5)(20) + (1+1+1) = 800 + 100 + 3 = 903 \\
|| ; \equiv ; : & = (10) (20\cdot 20) + 0 (20) + 6 = 4006
\end{align*}
\]

The process of converting from base 10 to Mayan is the same process as we used in the previous chapter. We will divide the base 10 number by 20, and continue dividing the quotient by 20 until the quotient is less than 20. We use the remainders and the final quotient in our numeral. Here are some examples.

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30Pascal, *Pensees*, p. 265
Example 2: Write 157 in Mayan.

Solution:

\[
\begin{array}{c}
20 \) 157 \\
140 \\
\hline
17
\end{array}
\]

So 157 = |.. ; |||..

Example 3: Write 1997 in Mayan.

Solution: …. ; | | …. ; | | ..

\[
\begin{array}{c}
20 \) 1997 \\
180 \\
\hline
197 \\
180 \\
\hline
17
\end{array}
\]

Mayan Addition

Addition in the common Mayan numeral system involves a variety of regrouping. Groups of 5 dots are replaced by a bar; groups of 20 must be “carried” to the next “column” as a one. Here are some examples:

Example 4: ... + |... = ||... [3 + 8 = 11]

Example 5: | |.. + || = .; .. [12 + 10 = 22]

Here the four five's (|| ||) equaling 20 have been carried into the 20's column as a one (.)

Example 6: | | ; .. + | | ; ||... = .; .. ; [207 + 233 = 440]

Solution: It might be easier to do problems like this vertically so that it is easier to see what happens in each column. I'll show two intermediate answers before final grouping and carrying occur.

\[
\begin{array}{c}
| | ; |.. \\
+ | | ; |... \\
| | | ; | | .. intermediate answer
\end{array}
\]

\[
| | | .. ; intermediate answer
\]

; .. ; final answer
Mayan Calendars

The Mayan culture took a very mathematical approach to time-cycles. For certain reasons, the Mayans maintained a calendar based on 13 "months" of 20 days each. Clearly, the resulting 260 day year would not work if your goal was to have seasons occur at the same time every year. For the purposes of matching the solar year, the Mayans had another calendar consisting of 18 "months" of 20 days each, with an additional 5 days at the end: a 365 day year! Every 52 years, the two calendars would start a new year on the same day. Sound slightly confusing? Well, apparently the Mayans were experts at converting dates between the two systems. We'll content ourselves with converting their numerals into ours.

When working with the solar calendar, the Mayans used a slight variation of the base twenty system. Since there were only 18 "months", the columns from the right were the 1's, the 20's, (and now the variation) the 18 x 20's, the 18 x 20^2 s, the 18 x 20^3 s, etc. That is, the third column was the 360's column, so a 3 column solar number could be understood as years, months, and days. To avoid as much confusion as possible with the common base twenty system described above, I will use a subscript "s" for the numerals in the Mayan solar calendar notation.

For instance, .. ; ... s would mean two 20's and three 1's, or 2 "months" of 20 days and 3 more days, i.e., 43 days. This looks just like the common Mayan system because the 1's column and the 20's column are the same in both systems. However, . ; .. ; ... s would mean one 360, two 20's, and three 1's, i.e., one year (the extra five days are being ignored), two months, and three days, i.e., 403 days.

Here are more examples:

Example 8:  |. .. ; |. |. ..| s  =  7 • 360 + 17 • 20 + 0  =  2,860

| |  ; |  |. |. s  =  10 • 360 + 1 • 20 + 16  =  3,636

To write a base ten number in Mayan solar often requires a slightly different process from the one we have been using to convert base ten numbers to a different base. You could think of the problem as converting a given number of days into years (360 days each), months (20 days each), and days. First we find out how many years there are, then how many months and days. For instance, convert 1038 to Mayan solar. Divide 1038 by 360 to determine how many years there are:

\[
\frac{1038}{360} = 2 \quad \text{remainder} \quad 238
\]

\[
\frac{238}{30} = 7 \quad \text{remainder} \quad 18
\]

\[
\frac{18}{20} = 0 \quad \text{remainder} \quad 18
\]

So, 1038 in Mayan solar is "2; 7; 18".
There are 2 years, with 318 days left over. Divide 318 by 20 to determine how many months and days there are:

\[
\begin{array}{c}
15 \\
20)318 \\
20 \\
118 \\
100 \\
18
\end{array}
\]

So \(1038 = \ldots ; | | ; | | \ldots\) -- that is, 2 years, 15 months, and 18 days.

**Example 9:** Write 547 in Mayan solar.

**Solution:**

\[
\begin{array}{c}
1 \\
360)547 \\
360 \\
187
\end{array}
\]

\[
\begin{array}{c}
9 \\
20)187 \\
20 \\
17
\end{array}
\]

547 = \ldots ; | | ; \ldots -- 1 year, 9 months, and 7 days.

**Example 10:** Write 3197 as Mayan solar.

**Solution:**

\[
\begin{array}{c}
8 \\
360)3197 \\
2880 \\
317
\end{array}
\]

\[
\begin{array}{c}
15 \\
20)317 \\
20 \\
100 \\
17
\end{array}
\]

3197 = \ldots ; | | ; | | \ldots -- 8 years, 15 months, and 17 days.

**Incas (1400-1560 A.D.)**

The Inca civilization has left us evidence of a highly organized bureaucracy with extensive record-keeping, but no writing either of words or numbers. The most significant evidence of the mathematical mind of the Incas is the quipus: spatial arrays of colored, knotted cords (as many as 2000). Apparently these elaborate arrays contain coded information based on sophisticated patterns and groupings. From the study of the quipus, it appears that the Incas used a base 10 positional numeral system.

**Native Americans of "California"**

The Native Americans who have lived in what we now call California used a variety of numeral systems. A common practice was to use a base five system for small numbers, but a base twenty system for larger numbers. This is similar to the grouping by 5 within a base 20 system used by the Mayans. Some other groups used a base ten system. One group, the Yukis, used a base 4 and a base 8 system. This may seem strange until one realizes the way they count on their hands. Instead of counting by using their fingers, they used the spaces between their fingers!
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Homework

1. Convert the following common Mayan numerals to base ten:
   a)  |||...
   b)  | ; ..
   c)  || ; .. ; ....
   d)  | .... ; | ; ..
   e)  | ; .. ; ...
   f)  .. ; | ; ...

2. Write the following base ten numbers as common Mayan numerals:
   a)  123
   b)  946
   c)  1994
   d)  480

3. Convert the following Mayan solar calendar numerals to base ten:
   a)  |.. ; ||| ; |....
   b)  || ; ||| ; ....
   c)  . ;
   d)  ... ; .. ; ||.

4. Write the following base ten numbers as Mayan solar calendar numerals:
   a)  123
   b)  946
   c)  1994
   d)  400

5. Perform the following additions:
   a)  || .. + | ....
   b)  || ; | .. + .... ; ||
   c)  || ; .. ; .... ;
   d)  | .. ; | .... + | .. ; .... ;

6. Perform the following additions:
   a)  || .. + | ...
   b)  || ; .... + | ... ; ||
   c)  || ; ||| ... ; |.... + | ...
   d)  ||| ; |.... + | .. ; |....

Answers:
1. a) 18      b) 135      c) 4449      d) 3712      e) 2003
2. a) . ; ...
     b) .. ; .. ; |
     c) . ; .... ;
3. a) 2829      b) 3695
4. a) . ; ..
     b) .. ; || ; ..
     c) | ; .... ; ||..
     d) . ; .. ; ..
5. a) . ; ..
     b) .. ; .. ; ..
     c) | || ... ; |...
6. a) . ; |
     b) ||| ... ; |||
     c) | || .... ; .. ;